cuts the circumference at C, then will ABC be the required triangle. For, since $DE=BD$, 2 angle $EBD=180^\circ-$angle $EDB=180^\circ-A$.

$\therefore$ Angle $EBD=90^\circ-\frac{1}{2}A$, but angle $EBA=$angle $EBD-$angle $ABD=$angle $EBD-\frac{1}{2}C=90^\circ-\frac{1}{2}A-\frac{1}{2}C=\frac{1}{2}B$.

$\therefore$ BE is the bisector of B, and by construction, CD is the bisector of C.

$\therefore$ E is the center of the inscribed circle. Q. E. D.

Also solved by *G. B. M. ZERR*, *P. S. BERG*, *COOPER D. SCHMITT*, *F. H. POWE*, *F. W. HAMAWALT*, *ELMER SCHUYLER*, and the *PROPOSER*.

CALCULUS.

81. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Instructor in Mathematics, Carthage High School, Carthage, Texas.

Solve : $y^2(d^2y/dx^2)+a(dy/dx)^2=bx$.

No solution of this problem has been received.

82. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

A pole 60 feet high stands vertically in a river 20 feet deep. How many feet above the surface of the water must it break so that the top bending down would touch the bottom and the distance on the surface of water between the points where the parts of the pole enter the water would be a maximum?

I. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O., and GUY B. COLLIER, Union College, B. S. Course, Schenectady, N. Y.

Let $x=$the number of feet above the surface of the water the pole must break, and $y=$the number of feet between the parts of the pole on the surface of the water, which is to be a maximum.

By similar triangles we find $y=\dfrac{2x}{x+20}\sqrt{300+30x}$.

Simplifying and placing the first derivative equal to zero, we have a biquadratic in x whose roots are : 0, -20, 6.055, and -66.055. By substitution in the second derivative we find that 6.055 is the only one of these roots that renders y a maximum. Therefore $x=6.055$ is the required result.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let ABC represent the pole, BC being the part under water. Let D be the point where it breaks off, so that $DA=DE$. Let $AB=a$, $BC=b$, $BD=x$, $BF=y$; then $DA=DE=a-x$. $CE=\sqrt{[(a-x)^2-(b+x)^2]}=\sqrt{(a+b)}.\sqrt{(a-b-2x)}$ and $CE:y=b+x:x$, whence $y=\sqrt{a+b}.\dfrac{x}{b+x}\sqrt{a-b-2x}$.

$\therefore M=\dfrac{x^2}{(b+x)^2}(a-b-2x)$ is to be a maximum.

By differentiation we obtain after all the necessary and simple transformations the quadratic $x^2+3bx=(a-b)b$, whence $x=\frac{1}{2}[-3b+\sqrt{(5b^2+4ab)}]$.

For the numerical value $a=40$, $b=20$, we get $x=10(\sqrt{13}-3)=6.055$.

III. **Solution by W. W. LANDIS, A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.; H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; O. S. WESTCOTT, A. M., Sc. D., Principal North Division High School, Chicago, Ill.; CHAS. E. MEYERS, Canton, Ohio; NELSON L. RORAY, Bridgetown, N. J.; CHAS. C. CROSS, Libertytown, Md.; ELMER SCHUYLER, High Bridge, N. J.; J. H. DRUMMOND, LL. D., Portland, Me.; COOPER D. SCHMITT, A. M. University of Tennessee, Knoxville, Tenn.; and the PROPOSER.**

Let x=distance above the water, y=the required maximum.

Then $40-x$ is the hypotenuse and $\sqrt{[(40-x)^2-(20+x)^2]}=2\sqrt{(300-30x)}$ =the base.

$\therefore\ y:x=2\sqrt{(300-30x)}:20+x.$

$\therefore\ y=\dfrac{2x\sqrt{(300-30x)}}{20+x}$=maximum.

Differentiating and reducing we get $(600-90x)(20+x)-2x(300-30x)=0$.

$\therefore\ x^2+60x=400.$

$\therefore\ x=6.0555128$ feet.

MECHANICS.

63. **Proposed by A. H. BELL, Hillsboro, Ill.**

From a horizontal support at a distance of 10 feet apart, a beam 5 feet long and 10 pounds weight is suspended by ropes attached to each end. The ropes are 3 and 5 feet respectively, in length. Required the angles made by the ropes and horizontal support. Also the stress upon each rope.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa., and the PROPOSER.

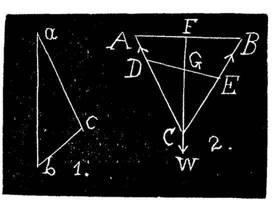
Let $AB=10$, $BC=CD=5$, $AD=3$, $EB=x+y$, $AE=x-y$.

$$\cos AEB=\frac{(x+y)^2+(x-y)^2-100}{2(x+y)(x-y)}=\frac{(x+y-5)^2+(x-y-3)^2-25}{2(x+y-5)(x-y-3)}.$$

$$\therefore\ (139-32x)y^2-(200-8x^2)y-79x^2+800x-1500=0\ \ldots\ldots\ \ldots\ldots\ \ldots(1).$$

$$DF=CN=\frac{100+(x-y)^2-(x+y)^2}{20(x-y)}(x-y-3)=\frac{100+(x+y)^2-(x-y)^2}{20(x+y)}(x+y-5).$$

$\therefore\ xy^3-xy^2-(x^3-4x^2+100)y+25x=0\ldots.(2).$



Let $139-32x=c$, $200-8x^2=2a$,
$79x^2-800x+1500=b$, $x^3-4x^2+100=d$.

$\therefore\ cy^2-2ay-b=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3),$

and $xy^3-xy^2-dy+25x=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$

y^2 of (3) in (4) $2axy^2+(bx-2ax-cd)y-bx+25cx=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5).$